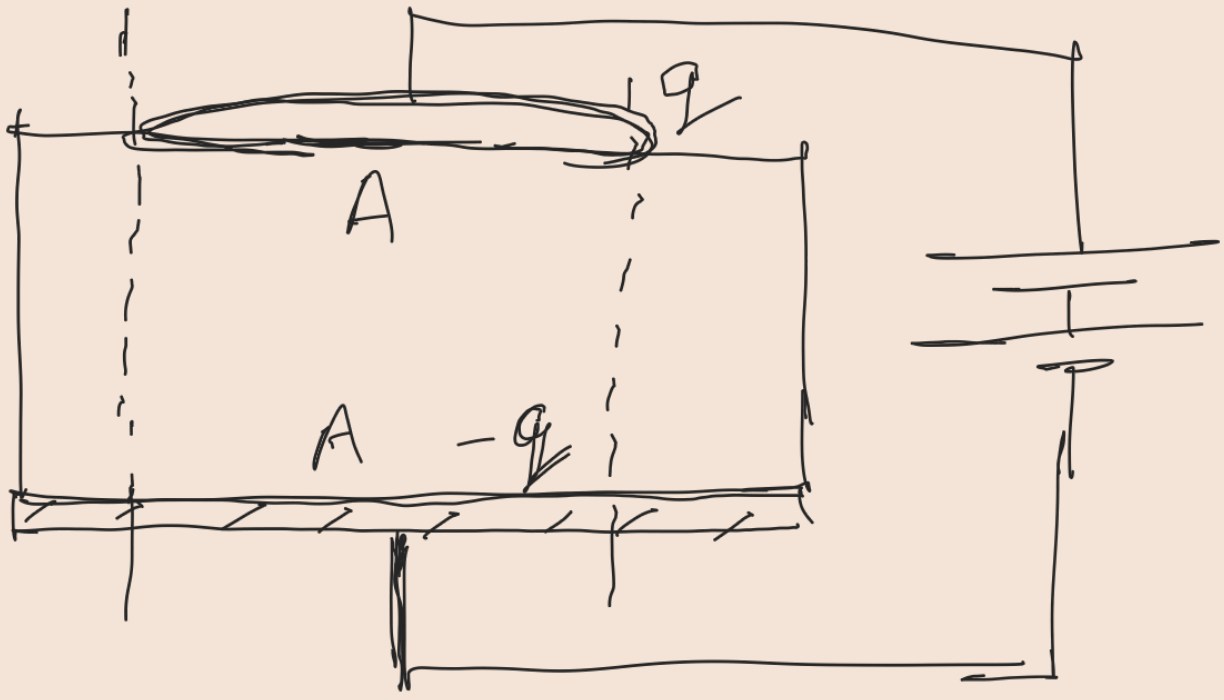


At any instant, let the surface area of the drop be 'A'. We note that drop is conducting & hence, the only charge residing area on both plates will be 'A'. Process is very slow & hence, there is

practically no heat loss.
Hence, at any general instant,

$$\frac{qd}{\epsilon_r A \epsilon_0} = V$$

$$q = VA\left(\frac{\epsilon_r \epsilon_0}{d}\right).$$

$$dq = \left(V dA + A dV\right)\left[\frac{\epsilon_r \epsilon_0}{d}\right]$$

Now, by work energy eqn,

$$V dq = dU_E + dU_{ST}.$$

$$U_E = \frac{1}{2}\frac{A\epsilon_0\epsilon_r}{d}V^2$$

$$dU_E = \frac{1}{2}\frac{\epsilon_0\epsilon_r}{d}\left(2V dV A + V^2 dA\right)$$

$$dU_{ST} = 2\Gamma dA. \quad \text{So,}$$

$$\left[V^2 dA + AV dV\right]\left(\frac{\epsilon_r \epsilon_0}{d}\right)$$

$$= \frac{\epsilon_0 \epsilon_r}{d}\left(V dV A + \frac{V^2 dA}{2}\right)$$

$$+ 2\Gamma dA$$

So,

$$\frac{V^2 \cancel{dA}}{2} \frac{\epsilon_0 \epsilon_r}{d} = 2\Gamma \cancel{dA}$$

$$\boxed{V = 2\sqrt{\frac{\Gamma d}{\epsilon_0 \epsilon_r}}}$$

. Two

very important points :-

(i) The 'Hg' drop has two surface areas & hence '2A' is taken for surface energy calculation.

(ii) We obtain a particular value of battery voltage irrespective of area. It means before the voltage, drop wont expand at all & beyond this, the drop automatically spreads out over entire surface.